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Bekenstein Bound, Holography and Brane Cosmology in Charged Black Hole Backgrounds

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Abstract

We obtain a Bekenstein entropy bound for the charged objects in arbitrary dimensions ($D \geq 4$) using the D-bound recently proposed by Bousso. With the help of thermodynamics of CFTs corresponding to AdS Reissner-Norström (RN) black holes, we discuss the relation between the Bekenstein and Bekenstein-Verlinde bounds. In particular we propose a Bekenstein-Verlinde-like bound for the charged systems. In the Einstein-Maxwell theory with a negative cosmological constant, we discuss the brane cosmology with positive tension using the Binetruy-Deffayet-Langlois approach. The resulting Friedman-Robertson-Walker equation can be identified with the one obtained by the moving domain wall approach in the AdS RN black hole background. Finally we also address the holographic property of the brane universe.

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1 Introduction

In a recent paper [1], Verlinde made two interesting observations. One is that according to the AdS/CFT correspondence, the entropy of a conformal field theory (CFT) in any dimension can be expressed in terms of a generalized form of the Cardy formula [2]. Consider a certain CFT residing in an $(n+1)$-dimensional spacetime with the metric

$$ds^2 = -dt^2 + R^2 d\Omega_n^2, \tag{1.1}$$

where $d\Omega_n^2$ denotes the line element of a unit n-dimensional sphere. It is proposed that the entropy of the CFT can be related to its total energy E and Casimir energy E_c as

$$S = \frac{2\pi R}{\sqrt{ab}} \sqrt{E_c(2E - E_c)}. \tag{1.2}$$

Here a and b are two positive parameters. For strongly coupled CFTs with the AdS duals, which implies that the CFTs are in the regime of supergravity duals, ab is fixed to be n^2 exactly. Thus one obtains the Cardy-Verlinde formula

$$S = \frac{2\pi R}{n} \sqrt{E_c(2E - E_c)}. \tag{1.3}$$

Indeed, this formula holds for various kinds of AdS spacetimes: AdS Schwarzschild black holes [1]; AdS Kerr black holes [3]; charged AdS black holes [4]; Taub-Bolt AdS spacetimes [5], whose thermodynamics corresponds to that of different CFTs [6].

The other comes from the comparison of the Cardy formula with the Friedman-Robertson-Walker (FRW) equation. For an $(n+1)$-dimensional closed universe, the FRW equations are

$$H^2 = \frac{16\pi G_n}{n(n-1)} \frac{E}{V} - \frac{1}{R^2}, \tag{1.4}$$

$$\dot{H} = -\frac{8\pi G_n}{n-1} \left(\frac{E}{V} + p \right) + \frac{1}{R^2}, \tag{1.5}$$

where $H = \dot{R}/R$ is the Hubble parameter, the dot stands for the differentiation with respect to the proper time, E is the total energy of matter filling in the universe, p denotes the pressure and $V = R^n \mathrm{Vol}(S^n)$ is the volume of the universe. In addition, G_n is the $(n+1)$-dimensional gravitational constant. Verlinde pointed out that the FRW equation (1.4) can be related to three cosmological entropy bounds:[1]

$$
\begin{aligned}
\text{Bekenstein-Verlinde bound}: &\quad S_{\rm BV} = \tfrac{2\pi}{n} ER, \\
\text{Bekenstein-Hawking bound}: &\quad S_{\rm BH} = (n-1)\tfrac{V}{4G_n R}, \\
\text{Hubble bound}: &\quad S_{\rm H} = (n-1)\tfrac{HV}{4G_n}.
\end{aligned}
\tag{1.6}
$$

The FRW equation (1.4) can then be rewritten as

$$
S_{\rm H} = \sqrt{S_{\rm BH}(2S_{\rm BV} - S_{\rm BH})}. \tag{1.7}
$$

The Bekenstein-Verlinde bound is valid for the weakly self-gravitating universe ($HR < 1$), while the Hubble bound holds for the strongly self-gravitating universe ($HR > 1$). It is clear from the FRW equation (1.4) that at the critical point of $HR = 1$, three entropy bounds coincide with each other.

Let us define $E_{\rm BH}$ corresponding to the Bekenstein-Hawking entropy bound by the Bekenstein-Verlinde bound such that $S_{\rm BH} = (n-1)V/4G_nR \equiv 2\pi E_{\rm BH}R/n$. The FRW equation (1.7) then takes the form

$$
S_{\rm H} = \frac{2\pi R}{n}\sqrt{E_{\rm BH}(2E - E_{\rm BH})}. \tag{1.8}
$$

This relation has the same form as the Cardy-Verlinde formula (1.3): the entropy S and the Casimir energy E_c of CFTs are replaced by the Hubble entropy bound S_H and the black hole mass $E_{\rm BH}$ corresponding to the Bekenstein-Hawking entropy bound. This means that the FRW equation somehow knows the entropy of CFTs filling in the universe [1]. This connection between the Cardy-Verlinde formula and the FRW equation can be interpreted as a consequence of the holographic principle [1].

More recently, Savonije and Verlinde [8] have studied the brane cosmology in the background of $(n+2)$-dimensional AdS Schwarzschild spacetimes. It turns out that the equations governing the motion of the brane are exactly the $(n+1)$-dimensional FRW equations with radiation matter. This radiation matter can be viewed as the CFT corresponding to the black hole in the AdS/CFT correspondence. In addition, it is found that the FRW equation is exactly matched with the Cardy-Verlinde formula for CFTs when the brane crosses the black hole horizon.

[1] In Ref. [1] the first bound is called the Bekenstein bound. In fact, this bound is slightly different from the original Bekenstein entropy bound [7]. So we call this the Bekenstein-Verlinde bound in our paper. We have more to comment on this point.

In this paper we extend this holographic connection to the AdS Reissner-Nordström (RN) black hole background in arbitrary dimensions. In the next section, we obtain a Bekenstein entropy bound for charged objects in arbitrary dimensions ($D \geq 4$). This will be derived by using the D-bound proposed by Bousso [9]. In Sec. 3, we briefly review thermodynamic aspects of the AdS RN black holes. We discuss the relation between the Bekenstein and Bekenstein-Verlinde bounds. In particular, we propose a Bekenstein-Verlinde-like bound for charged systems.

In Sec. 4 we consider the cosmology of brane with positive tension in the Einstein-Maxwell theory with a negative cosmological constant using the Binetruy-Deffayet-Langlois (BDL) approach [10, 11]. We find that the resulting FRW equation can be identified with that obtained by Biswas and Mukherji [12] by using the moving domain wall approach [13, 14, 15]. Finally we address the holographic property of this brane cosmology. The conclusions are given in Sec. 5.

Other related discussions to Verlinde's observations can be found in Refs. [16]-[24].

2 Bekenstein bound in arbitrary dimensions

Bekenstein [7] is the first to consider the issue of maximal entropy for a macroscopic system. He argued that for a closed system with total energy E, which fits in a sphere with radius R in three spatial dimensions, there exists an upper bound on the entropy

$$S \leq S_{\rm B} = 2\pi RE. \tag{2.1}$$

This is called the Bekenstein entropy bound.

The Bekenstein bound is believed to be valid for a system with the limited self-gravity, which means that the gravitational self-energy is negligibly small compared to its total energy. However, it is interesting to note that the bound (2.1) is saturated even for a four-dimensional Schwarzschild black hole which is a strongly self-gravitating object. Furthermore, it has been found in Ref. [9] that the form of the Bekenstein bound (2.1) is independent of the spatial dimensions. This was obtained by considering the Geroch process in a $D(\geq 4)$-dimensional Schwarzschild background and the generalized second law of black hole thermodynamics. It implies that the Bekenstein bound in arbitrary dimensions ($D \geq 4$) always remains in the same form (2.1). It is easy to check that a $D(\geq 4)$-dimensional Schwarzschild black hole satisfies the bound (2.1). But the bound will no longer be saturated for $D > 4$.

The bound (2.1) has been extended recently to the case of charged objects in four dimensions [25, 26, 27, 28]. It is found that the Bekenstein bound is modified to

$$S \leq S_{\rm B} = \pi(2ER - Q^2), \tag{2.2}$$

for a closed system with charge Q. This bound is saturated by a four-dimensional RN black hole with mass E and charge Q.

It is very interesting to investigate whether or not the form (2.2) of the Bekenstein bound for a charged object in arbitrary dimensions ($D \geq 4$) remains unchanged as in the case for the neutral objects. Furthermore, we need such a bound in order to discuss the holographic aspects of the brane universe in the AdS RN black hole background. An important ingredient in deriving the Bekenstein bound (2.2) is the electrostatic self-energy of a charged test object in a black hole background [25, 26, 27, 28]. However, it is not easy to obtain this quantity in higher dimensions. So here we use the D-bound proposed by Bousso [9] to get a Bekenstein bound for a charged system in arbitrary dimensions.

We start with the $(n+2)$-dimensional Einstein-Maxwell theory with a cosmological constant $\Lambda_\pm = \pm n(n+1)/2l^2$:

$$I = \frac{1}{16\pi \mathbf{G_n}} \int d^{n+2}x \sqrt{-g}\left(\mathcal{R} - F_{\mu\nu}F^{\mu\nu} - 2\Lambda_\pm\right), \tag{2.3}$$

where $\mathcal{R}$ is the curvature scalar, F denotes the Maxwell field, and $\mathbf{G_n}$ is the gravitational constant in $(n+2)$ dimensions. Varying the action (2.3) yields the equations of motion

$$G_{\mu\nu} \equiv \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = T_{\mu\nu}, \quad T_{\mu\nu} = 2F_{\mu\lambda}F_\nu{}^\lambda - \frac{1}{2}g_{\mu\nu}F^2 - \Lambda_\pm g_{\mu\nu}, \tag{2.4}$$

$$\partial_\mu(\sqrt{-g}F^{\mu\nu}) = 0. \tag{2.5}$$

These equations have a spherically symmetric solution [29, 30]

$$\begin{aligned} ds^2 &= -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_n^2, \\ F_{rt} &= \frac{n\omega_n}{4}\frac{Q}{r^n}, \qquad \omega_n = \frac{16\pi\mathbf{G_n}}{n\mathrm{Vol}(S^n)}, \end{aligned} \tag{2.6}$$

where $\mathrm{Vol}(S^n)$ is the volume of a unit n-sphere and the function f is given by

$$f_\pm(r) = 1 - \frac{\omega_n M}{r^{n-1}} + \frac{n\omega_n^2 Q^2}{8(n-1)r^{2(n-1)}} - \frac{2\Lambda_\pm r^2}{n(n+1)}. \tag{2.7}$$

This solution is asymptotically de Sitter (dS) or anti-de Sitter (AdS) depending on the cosmological constant Λ_+ or Λ_-. In this section we consider the de Sitter case.

We note that if $M = Q = 0$, the solution reduces to the dS space which has a cosmological horizon at $r = r_0 \equiv \sqrt{l^2}$. The cosmological horizon behaves in many aspects like the black hole horizon [31]. In particular, it has the thermodynamic entropy

$$S_0 = \frac{r_0^n}{4\mathbf{G_n}}\mathrm{Vol}(S^n). \tag{2.8}$$

In a more general case with nonvanishing M and Q, the solution describes the geometry of a certain object with mass M and electric charge Q embedded in dS space[2]. The cosmological horizon will shrink due to the nonzero M and Q. This leads to the N-bound of Bousso [9]. This bound claims that in the asymptotically dS spacetime, the maximally observable degrees of freedom are bounded by the entropy (2.8) of the exact dS space. Furthermore, applying the Geroch process to the cosmological horizon leads to the D-bound in dS space [9]. This tells us that the entropy of objects in dS space is bounded by the difference of the entropies in the exact dS space and in the asymptotically dS space

$$S_m \leq S_0 - S_c, \tag{2.9}$$

where S_c is the cosmological horizon entropy when matter is present.

Let us apply this D-bound to the dS RN spacetime (2.6). Here the cosmological horizon r_c is given by the maximal root of the equation:

$$1 - \frac{\omega_n M}{r_c^{n-1}} + \frac{n\omega_n^2 Q^2}{8(n-1)r_c^{2(n-1)}} - \frac{r_c^2}{r_0^2} = 0. \tag{2.10}$$

This leads to

$$\frac{r_0^n}{r_c^n} = \left(1 - \frac{\omega_n M}{r_c^{n-1}} + \frac{n\omega_n^2 Q^2}{8(n-1)r_c^{2(n-1)}}\right)^{-n/2}. \tag{2.11}$$

Consider the large cosmological horizon limit of $M/r_c^{n-1} \ll 1$ and $Q^2/r_c^{2(n-1)} \ll 1$, we have

$$\frac{r_0^n}{r_c^n} \approx 1 + \frac{n\omega_n M}{2r_c^{n-1}} - \frac{n^2\omega_n^2 Q^2}{16(n-1)r_c^{2(n-1)}}, \tag{2.12}$$

in the leading order of M/r_c^{n-1} and $Q^2/r_c^{2(n-1)}$. Substituting the above to the D-bound (2.9) gives

$$\begin{aligned} S_m &\leq \frac{\mathrm{Vol}(S^n)}{4\mathbf{G_n}}(r_0^n - r_c^n) \\ &\leq 2\pi r_c\left(M - \frac{2\pi\mathbf{G_n}Q^2}{(n-1)\mathrm{Vol}(S^n)r_c^{n-1}}\right). \end{aligned} \tag{2.13}$$

[2] In asymptotic dS space, the energy of the system is not well-defined due to the absence of a suitable spacelike infinity. Here the constant M is viewed as the mass of the object in the sense of Ref. [32].

One finds that the entropy reaches its maximum when the matter extends to the cosmological horizon since the distribution range of matter is bounded by the cosmological horizon. Replacing r_c by R and M by the proper energy E, we get an entropy bound of the charged object in arbitrary dimensions ($D = n + 2 \geq 4$):

$$S \leq S_{\rm B} = 2\pi R \left(E - \frac{2\pi \mathbf{G_n} Q^2}{(n-1)V(S^n)R^{n-1}} \right). \tag{2.14}$$

To check this result (2.14), we note that when $Q^2 = 0$, this bound reproduces precisely the Bekenstein bound (2.1) for the neutral object in arbitrary dimensions. For $n = 2$, the entropy bound (2.14) reduces to the four-dimensional one (2.2).[3] Furthermore, comparing (2.14) and (2.2), we note that unlike the neutral case, the Bekenstein bound for charged objects in higher dimensions ($D > 4$) changes its form from that in four dimensions. In addition, one can see that the entropy bound (2.14) will no longer be saturated by a higher ($D > 4$) dimensional RN black hole. This is the same as the case of neutral objects.

We can rewrite the bound (2.14) in a more holographic form:

$$S_{\rm B} = 2\pi R \left(E - \frac{2\pi \mathbf{G_n} R Q^2}{(n-1)\mathcal{A}} \right), \tag{2.15}$$

where $\mathcal{A} = R^n \text{Vol}(s^n)$ is the surface area of the charged object. The second term in the r.h.s. of Eq. (2.15) can be understood as the energy E_q of the electromagnetic field outside the charged object. Hence one can also write

$$S_{\rm B} = 2\pi R(E - E_q), \quad E_q = \frac{1}{2}\phi Q, \tag{2.16}$$

where $\phi = \frac{n\omega_n}{4(n-1)} \frac{Q}{R^{n-1}}$ is the electrostatic potential at the surface of the charged object. In deriving Eq. (2.16), we have assumed that the potential vanishes at the spatial infinity.

3 AdS Reissner-Nordström black holes and Bekenstein-Verlinde bound

In this section we consider the AdS case of Λ_- in the solution (2.6). The cosmological horizon is absent in this case and the solution (2.6) describes the AdS RN black hole in

[3] In the bound (2.2), the gravitational constant $\mathbf{G_n}$ is absent, but it appears in (2.14). This is due to the different units for electric charge used in [25, 26, 27, 28] and in this paper.

arbitrary dimensions. The black hole horizon r_+ is determined by the maximal root of the equation $f_-(r_+) = 0$. The integration constants M and Q can be interpreted as the mass and electric charge of the black hole.

In the spirit of the AdS/CFT correspondence, the thermodynamics of AdS RN black holes corresponds to that for the boundary CFT with an R-charge (or R-potential). In Ref. [4], it was shown that indeed the entropy of the corresponding CFTs can be expressed in terms of the Cardy-Verlinde form (1.3). In this section we further discuss aspects of the thermodynamic properties and suggest a Bekenstein-Verlinde-like bound for the charged systems.

We rescale the boundary metric so that it has the form (1.1). Thermodynamic quantities of the corresponding CFT are given by

$$\begin{aligned}
E &= \frac{l r_+^{n-1}}{R\omega_n}\left(1 + \frac{r_+^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_+^{2(n-1)}}\right), \\
T &= \frac{l}{4\pi R r_+}\left((n-1) + \frac{(n+1)r_+^2}{l^2} - \frac{n\omega_n^2 Q^2}{8 r_+^{2(n-1)}}\right), \\
\Phi &= \frac{n l \omega_n Q}{4(n-1) R r_+^{n-1}}, \\
S &= \frac{r_+^n}{4\mathbf{G_n}}\mathrm{Vol}(S^n), \\
G &= \frac{l r_+^{n-1}}{nR\omega_n}\left(1 - \frac{r_+^2}{l^2} - \frac{n\omega_n^2}{8(n-1)}\frac{Q^2}{r_+^{2(n-1)}}\right),
\end{aligned} \tag{3.1}$$

where r_+ is the horizon of the AdS RN black hole, and E, T, Φ, S and G represent the energy, temperature, chemical potential conjugate to the charge Q, entropy and Gibbs free energy of the CFT, respectively. In analogy to the Cardy-Verlinde formula (1.3), we can rewrite the entropy in Eq. (3.1) as

$$S = \frac{2\pi R}{n}\sqrt{E_c(2(E - E_q) - E_c)}, \tag{3.2}$$

where

$$E_c = \frac{2 l r_+^{n-1}}{\omega_n R}, \quad E_q = \frac{1}{2}\Phi Q = \frac{l}{R}\frac{n\omega_n}{8(n-1)}\frac{Q^2}{r_+^{n-1}}. \tag{3.3}$$

We note that E_q is the bulk energy of electromagnetic field $E_B = -\frac{1}{16\pi \mathbf{G_n}}\int_{r_+}^{\infty} d^{n+2}x\sqrt{-g}F^2$ multiplied by the scale factor (l/R). For the fixed E, R and E_q, we find from (3.2) that the entropy reaches its maximal value

$$S_{\max} = \frac{2\pi R}{n}(E - E_q)$$

$$= \frac{2\pi}{n}\left(ER - \frac{nl\omega_n}{8(n-1)}\frac{Q^2}{r_+^{n-1}}\right), \tag{3.4}$$

at $E_c = E - E_q$. Note that the above expression is quite similar to the Bekenstein bound (2.16) for the charged objects.

Now let us discuss the difference and relation between the Bekenstein-Verlinde bound $S_{\rm BV}$ in (1.6) and the Bekenstein bound (2.1). In the AdS/CFT correspondence, the CFT resides in the UV boundary of the AdS spacetime and the gravity decouples on the boundary. Recall that the Cardy-Verlinde formula (1.3) is supposed to give the entropy of the $(n+1)$-dimensional CFT residing in the spacetime (1.1), UV boundary of the AdS space. It holds at least in the regime of supergravity duals. Note further that the Cardy-Verlinde formula (1.3) gives the maximal entropy ($S = 2\pi ER/n$), when the Casimir energy equals the total energy ($E_c = E$). This is just the Bekenstein-Verlinde bound $S_{\rm BV}$. Therefore it is reasonable to regard the Bekenstein-Verlinde bound $S_{\rm BV}$ as the maximal entropy bound of CFTs in (1.1), rather than a certain entropy bound of a system with gravity. On the other hand, the Bekenstein bound (2.1) for neutral objects holds for a closed system in an asymptotically spacetime. Therefore, the Beksentein bound and the Bekenstein-Verlinde bound are applicable in different spacetimes. Furthermore, the Bekenstein bound for neutral objects remains in the same form (2.1) in any dimensions [9], while the form of the Bekenstein-Verlinde bound depends on the spacetime dimension (n).

Let us consider the brane world scenario [33] in the generalized AdS/CFT correspondence. The gravity does not decouple on the brane because the brane is not on the UV boundary of AdS space but it is located in the bulk of the AdS space. Like the Bekenstein bound (2.1), suppose the Bekenstein-Verlinde bound in (1.6) is also valid for CFTs with limited self-gravity. Thus we can regard the Bekenstein-Verlinde bound $S_{\rm BV}$ in (1.6) as a counterpart (on the brane) of the Bekenstein bound (2.1), because the reduced metric on the brane is of the form (1.1). That is, the Bekenstein bound is valid in the bulk, while the Bekenstein-Verlinde bound holds on the brane. Of course, for both cases, the gravity is assumed to be weak.

The FRW cosmology renders a piece of evidence for supporting this. We know that the Bekenstein bound (2.1) cannot be naively used for a closed universe because of the lack of a suitable boundary [1]. On the other hand, the Bekenstein-Verlinde bound holds in the spacetime (1.1). This has a scale R, which can be naturally identified with the cosmic scale R in the FRW cosmology. Furthermore, the Bekenstein-Verlinde bound $S_{\rm BV}$

appears in the FRW equation. Thus, we can also view the Bekenstein-Verlinde bound as a counterpart of the Bekenstein bound in the context of brane cosmology.

With these considerations, we now suggest a Bekenstein-Verlinde-like bound for a charged system. According to the AdS/CFT correspondence, the boundary spacetime in which the boundary CFT resides can be determined from the bulk metric, up to a conformal factor. The conformal factor enables us to rescale the boundary metric as we wish. In order to obtain a suitable bound, we rescale the boundary metric so that the radius R in (1.1) becomes the horizon radius r_+ of the black hole, as in [1]. The maximal entropy (3.4) is then given by

$$S_{\rm max} = \frac{2\pi R}{n}\left(E - \frac{nl\omega_n}{8(n-1)}\frac{Q^2}{R^n}\right). \tag{3.5}$$

Consider further the similarity between the Bekenstein bound (2.1) for neutral objects and the Bekenstein-Verlinde bound $S_{\rm BV}$ in (1.6), and also the similarity between the Bekenstein bound (2.15) for charged objects and the maximal entropy (3.4) of the CFT with R-charge. We propose the maximal entropy (3.5) as the Bekenstein-Verlinde-like bound $S_{\rm BV}$ for the CFT with the R-charge Q in the spacetime (1.1). We can rewrite the above as

$$S_{\rm BV} = \frac{2\pi R}{n}\left(E - \frac{2\pi \mathbf{G_n} l}{(n-1)}\frac{Q^2}{V}\right), \tag{3.6}$$

where $V = R^n \mathrm{Vol}(S^n)$. One may wonder why the bulk parameter l appears in the above bound. This can be understood by recalling that in the AdS/CFT correspondence, the cosmological constant is related to the 't Hooft coupling constant in the CFT. Furthermore, we will see that the bound (3.6) plays the same role in the brane cosmology in the AdS RN black hole background as $S_{\rm BV}$ in (1.6) in the FRW equation (1.7).

4 Brane cosmology in the charged background

Recently the cosmology of the Randall-Sundrum (RS) scenario [34] for a positive tension brane in a five-dimensional universe (with localized gravity) has been studied extensively. In most of works a negative cosmological constant is introduced as the bulk matter without else. In this section we consider a higher dimensional cosmology of RS scenario with a Maxwell field as well as the negative cosmological constant as the bulk matter. That is, we consider a brane universe in the bulk with the action (2.3).

Following BDL [10, 11], we assume the bulk metric is of the form

$$ds^2 = -c^2(t,y)dt^2 + a^2(t,y)\gamma_{ij}dx^i dx^j + b^2(t,y)dy^2, \tag{4.1}$$

where γ_{ij} is the metric of an n-dimensional space with constant curvature $n(n-1)k$. One may take $k = 1$, 0 and -1. In the orthogonal basis, we work out the Einstein tensor

$$\begin{aligned}
G_{\hat{t}\hat{t}} &= n\left[\frac{\dot{a}}{ac^2}\left(\frac{n-1}{2}\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right)-\frac{1}{b^2}\left(\frac{a''}{a}+\frac{a'}{a}\left(\frac{n-1}{2}\frac{a'}{a}-\frac{b'}{b}\right)\right)+\frac{n-1}{2}\frac{k}{a^2}\right],\\
G_{\hat{y}\hat{y}} &= n\left[\frac{a'}{ab^2}\left(\frac{n-1}{2}\frac{a'}{a}+\frac{c'}{c}\right)-\frac{1}{c^2}\left(\frac{\ddot{a}}{a}+\frac{\dot{a}}{a}\left(\frac{n-1}{2}\frac{\dot{a}}{a}-\frac{\dot{c}}{c}\right)\right)-\frac{n-1}{2}\frac{k}{a^2}\right],\\
G_{\hat{t}\hat{y}} &= n\left(\frac{\dot{a}c'}{abc^2}+\frac{a'\dot{b}}{ab^2c}-\frac{\dot{a}'}{abc}\right),\\
G_{\hat{i}\hat{j}} &= \frac{\delta_{ij}}{b^2}\left[(n-1)\frac{a''}{a}+\frac{c''}{c}+\frac{n-1}{2}\frac{a'}{a}\left((n-2)\frac{a'}{a}+2\frac{c'}{c}\right)-\frac{b'}{b}\left((n-1)\frac{a'}{a}+\frac{c'}{c}\right)\right]\\
&\quad+\frac{\delta_{ij}}{c^2}\left[-(n-1)\frac{\ddot{a}}{a}-\frac{\ddot{b}}{b}+\frac{\dot{b}}{b}\left(\frac{\dot{c}}{c}-(n-1)\frac{\dot{a}}{a}\right)+\frac{n-1}{2}\frac{\dot{a}}{a}\left(2\frac{\dot{c}}{c}-(n-2)\frac{\dot{a}}{a}\right)\right]\\
&\quad-\frac{(n-1)(n-2)}{2}\frac{k}{a^2}\delta_{ij},
\end{aligned} \tag{4.2}$$

where the dot (prime) stands for the differentiation with respect to t (y).

Now suppose that an n-dimensional brane is located at $y = 0$. On the two sides of the brane, the stress-energy tensors are given by (2.4), but they are not necessarily identified. For example, the cosmological constant may be different on two sides [35]. The stress-energy tensor on the brane is assumed to be of the form

$$\tau_\mu{}^\nu = \frac{\delta(y)}{b}\mathrm{diag}(-\rho, p, \cdots, p, 0), \tag{4.3}$$

which implies that the brane is homogeneous and isotropic.

Let us denote the gap of a given function f at $y = 0$ by $[f] = f(0_+) - f(0_-)$ and its average by $\{f\} = (f(0_+) + f(0_-))/2$. The functions a, b, c in the metric (4.1) are continuous at $y = 0$, but their derivatives are discontinuous. So the second derivatives are of the form [35]

$$f'' = f''|_{(y\neq 0)} + [f']\delta(y). \tag{4.4}$$

It is then straightforward to write down the gaps in the (tt), (yy) and (ij) components of the Einstein equation (2.4), respectively:

$$\frac{n}{b_0^2}\left(-(n-1)\frac{[a']\{a'\}}{a_0^2}+\frac{[a']\{b'\}}{a_0 b_0}+\frac{\{a'\}[b']}{a_0 b_0}\right) = T_{\hat{t}\hat{t}}(0_+) - T_{\hat{t}\hat{t}}(0_-),$$

$$\frac{n}{b_0^2}\left((n-1)\frac{[a']\{a'\}}{a_0^2}+\frac{[a']\{c'\}}{a_0c_0}+\frac{\{a'\}[c']}{a_0c_0}\right)=T_{\hat{y}\hat{y}}(0_+)-T_{\hat{y}\hat{y}}(0_-),$$

$$\frac{(n-1)}{b_0^2}\delta_{ij}\left((n-2)\frac{[a']\{a'\}}{a_0^2}+\frac{[a']\{c'\}}{a_0c_0}+\frac{\{a'\}[c']}{a_0c_0}-\frac{[a']\{b'\}}{a_0b_0}-\frac{\{a'\}[b']}{a_0b_0}\right.$$
$$\left.-\frac{1}{n-1}\frac{[b']\{c'\}}{b_0c_0}-\frac{1}{n-1}\frac{\{b'\}[c']}{b_0c_0}\right)=T_{\hat{i}\hat{j}}(0_+)-T_{\hat{i}\hat{j}}(0_-). \tag{4.5}$$

where the quantities with subscript 0 denote those at $y=0$. The δ-function parts in the (tt) and (ij) components of the Einstein equation give

$$\frac{[a']}{a_0b_0}=-\frac{8\pi\mathbf{G_n}}{n}\rho,\qquad \frac{[c']}{b_0c_0}=8\pi\mathbf{G_n}\left(p+\frac{n-1}{n}\rho\right). \tag{4.6}$$

On the other hand, the average part of the (yy)-component is

$$\frac{1}{c_0^2}\left(\frac{\ddot{a}_0}{a_0}+\frac{\dot{a}_0}{a_0}\left(\frac{n-1}{2}\frac{\dot{a}_0}{a_0}-\frac{\dot{c}_0}{c_0}\right)\right)=-\frac{1}{2n}\left(T_{\hat{y}\hat{y}}(0_+)+T_{\hat{y}\hat{y}}(0_-)\right)$$
$$+\frac{n-1}{2}\left(-\frac{k}{a_0^2}+\frac{1}{4}\left(\frac{[a']}{a_0b_0}\right)^2+\left(\frac{\{a'\}}{a_0b_0}\right)^2\right)+\frac{1}{4}\frac{[a'][c']}{a_0b_0^2c_0}+\frac{\{a'\}\{c'\}}{a_0b_0^2c_0}. \tag{4.7}$$

We note that the Maxwell equation (2.5) in the metric (4.1) has the solution

$$F_{yt}=\frac{\mathcal{Q}bc}{a^n}, \tag{4.8}$$

where $\mathcal{Q}$ is an integration constant. Thus the bulk stress-energy tensor including the Maxwell field and cosmological constant Λ_- is

$$T_{\hat{\mu}\hat{\nu}}=\mathrm{diag}\left(-\frac{n(n+1)}{2l^2}+\frac{\mathcal{Q}^2}{a^{2n}},\frac{n(n+1)}{2l^2}+\frac{\mathcal{Q}^2}{a^{2n}},\cdots,\frac{n(n+1)}{2l^2}+\frac{\mathcal{Q}^2}{a^{2n}},\frac{n(n+1)}{2l^2}-\frac{\mathcal{Q}^2}{a^{2n}}\right). \tag{4.9}$$

Now we consider a simple case in which the bulk is Z_2-symmetric and the bulk stress-energy tensors are identical on two sides of the brane. One then has $\{f'\}=0$. We define the cosmic time τ as $d\tau=c(t,0)dt$ and the Hubble parameter H on the brane at $y=0$ as $H=\dot{a}_0/a_0=\dot{R}/R$. Following [35], the l.h.s. of Eq. (4.7) can be rewritten as

$$\frac{1}{c_0^2}\left(\frac{\ddot{a}_0}{a_0}+\frac{\dot{a}_0}{a_0}\left(\frac{n-1}{2}\frac{\dot{a}_0}{a_0}-\frac{\dot{c}_0}{c_0}\right)\right)=\frac{1}{2R^n}\frac{d}{dR}H^2R^{n+1}, \tag{4.10}$$

while the r.h.s. can be calculated with the help of Eqs. (4.6) and (4.9). Then (4.7) leads to

$$\frac{1}{2R^n}\frac{d}{dR}H^2R^{n+1}=-\frac{(8\pi\mathbf{G_n})^{\mathbf{2}}}{8n^2}\rho\left(2np+(n-1)\rho\right)-\frac{n+1}{2l^2}-\frac{n-1}{2}\frac{k}{R^2}+\frac{\mathcal{Q}^2}{nR^{2n}}. \tag{4.11}$$

Without the localized matter on the brane, we have $p = -\rho = -n/4\pi l\mathbf{G_n}$. Here the tension of the brane is assumed to be fine-tuning. Thus Eq. (4.11) reduces to

$$\frac{d}{dR}H^2R^{n+1} = -(n-1)kR^{n-2} + \frac{2\mathcal{Q}^2}{nR^n}. \tag{4.12}$$

Integrating this equation yields

$$H^2 = \frac{\mathcal{C}}{R^{n+1}} - \frac{k}{R^2} - \frac{2}{n(n-1)}\frac{\mathcal{Q}^2}{R^{2n}}, \tag{4.13}$$

where $\mathcal{C}$ is an integration constant, which already appears in [11]. This is our equation derived from the BDL approach and governs the evolution of the brane universe in the bulk with a Maxwell field.

Now let us compare our equation (4.13) with the equation of motion for the moving domain wall (brane) in the AdS RN black hole solutions (2.6). The latter is [12]

$$H^2 = \frac{\omega_n M}{R^{n+1}} - \frac{1}{R^2} - \frac{n\omega_n^2 Q^2}{8(n-1)R^{2n}}. \tag{4.14}$$

Taking $k = 1$ and

$$\mathcal{C} = \omega_n M, \qquad \mathcal{Q}^2 = \frac{n^2\omega_n^2 Q^2}{16}, \tag{4.15}$$

one can see immediately that our equation (4.13) exactly coincides with Eq. (4.14). For Eq. (4.14) of the moving brane, the bulk is the AdS RN black hole geometry with the mass M and electric charge Q. These two parameters encode the information of the bulk and describe the CFT on the brane. On the other hand, in deriving Eq. (4.13), we do not have to know exactly what is the bulk geometry but we only have to know that there exists a Maxwell field in the bulk. However, the two integration constants $\mathcal{C}$ and $\mathcal{Q}$ certainly encode the information of the bulk. As a result, just as in the moving brane approach, in which the brane moves in the bulk and acts as the boundary of the bulk, the FRW equation of the brane in the BDL approach with fixed brane in the bulk, also encodes the information of the bulk, showing the same holographic property.

There is a relation between the gravitational constants in the bulk and on the brane [33, 8]:

$$\mathbf{G_n} = \frac{G_n l}{n-1}. \tag{4.16}$$

Relating the mass M of black hole with the energy on the brane, one can rewrite (4.14) as

$$H^2 = -\frac{1}{R^2} + \frac{16\pi G_n}{n(n-1)}\frac{E}{V} - \frac{32\pi^2\mathbf{G_n}G_n l}{n(n-1)^2}\frac{Q^2}{V^2}, \tag{4.17}$$

where $E = lM/R$ and $V = R^n\mathrm{Vol}(S^n)$. With the Bekenstein-Hawking bound $S_{\rm BH}$, Hubble bound $S_{\rm H}$ in (1.6) and the Bekenstein-Verlinde-like bound (3.6) we proposed in the previous section, we find that the FRW equation (4.17) can be expressed as

$$S_{\rm H} = \sqrt{S_{\rm BH}(2S_{\rm BV} - S_{\rm BH})}, \tag{4.18}$$

which is the same as (1.7), although the $S_{\rm BV}$ in (4.18) is different from the one in (1.7). With the Bekenstein-Hawking energy $E_{\rm BH}$ defined in (1.8, Eq. (4.18) can be further cast into

$$S_{\rm H} = \frac{2\pi R}{n}\sqrt{E_{\rm BH}\left(2\left(E - \frac{2\pi l\mathbf{G_n}}{n-1}\frac{Q^2}{V}\right) - E_{\rm BH}\right)}. \tag{4.19}$$

This is of the same form as the Cardy-Verlinde formula (3.2) for the CFT with an R-charge corresponding to the AdS RN black holes. In particular, when the brane crosses the horizon of AdS RN black holes, *i.e.*, $R = r_+$, Eq. (4.19) matches with the Cardy-Verlinde formula (3.2) exactly.

5 Conclusions

Using the D-bound of Bousso we have obtained a Bekenstein entropy bound (2.14) for charged systems in arbitrary dimensions ($D \geq 4$). When the charge vanishes, the bound reduces to the usual Bekenstein bound for neutral objects, while if one puts $D = 4$, it precisely reproduces the known Bekenstein entropy bound for charged objects in four dimensions. The Bekenstein entropy bound (2.14) is saturated by a four dimensional RN black hole. But as the case of neutral objects, the Bekenstein bound will no longer be saturated by higher ($D > 4$) dimensional RN black holes.

We have also discussed the difference and relation between the Bekenstein and Bekenstein-Verlinde bounds, and argued that the Bekenstein-Verlinde bound could be regarded as the counterpart of the Bekenstein bound in the context of cosmology. With the thermodynamics of AdS RN black holes, a Bekenstein-Verlinde-like bound for charged system has been suggested. In addition, we have studied the brane cosmology of RS scenario in the Einstein-Maxwell theory with a negative cosmological constant in the BDL approach. The resulting FRW equation can be identified with the one which governs the motion of a domain wall in the AdS RN black hole background. With the Bekenstein-Verlinde-like bound and others, our FRW equation can be cast into the form of Cardy-Verlinde formula, which describes the entropy of CFT with an R-charge filling the brane universe.

Our results further indicate that the brane cosmologies resulting from the BDL approach and from the moving brane approach can be identified with each other, and show the same holographic properties in the case of Einstein-Maxwell theory. This paper also supports and extends the result of [36]. There the authors discussed the relation between the BDL approach and moving domain wall approach in the case where the bulk has only a negative cosmological constant.

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